



AF-TR-9400-001

Quantitative Methods and Worked Calculations

Space / Aviation / Defense

FIRST EDITION

AF 9400 SUPPORTING DOCUMENT

Document identity and use

FIRST EDITION MANUSCRIPT - Prepared for Aerospace Federation review

Field	Controlled identity
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Parent standard	AF 9400 - Aerospace Manufacturing, Materials & Supply Chain
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Primary source anchors	10, 13-14, 25-26, 34; informative Annex O
Suggested local owner	Metrology / statistics / technical specialist
Authority	Publisher approval and effective date are not assigned. Local use requires authorized adoption and completed parameters.

Use with the main standard

This document implements the identified AF 9400 clauses and annexes. Read the complete applicable requirements in the main standard. The source anchors are navigation references; a template does not replace technical judgement, evidence, permission or eligible approval.

AF-ML-9400-001 lists the complete supporting set. AF-CM-9400-001 maps the main clauses and annexes to working documents. AF-TP-9400-001 establishes the common local parameter baseline; add the relevant discipline profiles.

Record control

Identify the object, configuration, record owner and dates. Reference attachments by stable identifier and version. Keep superseded evidence and decisions traceable. Use the approved local retention, access and authentication arrangements. No process recipe, numerical limit, interval or acceptance value is assigned by a blank template.

Implementation model: Translate the stated responsibilities and controls into the actual organization. Informative examples remain explanatory; use approved programme-specific values for operational decisions.

1 Use of quantitative examples

All values and cases in this companion are synthetic. They explain assumptions, calculations and limitations; they are not process limits, product evidence or mandatory default parameters. Select and approve the actual model and values before relying on a calculation.

2 Worked calculations

Status: Informative. Every value, population and outcome in this annex is synthetic. These examples explain methods and limitations; they are not material allowables, test evidence, mandatory targets or records of actual production.

0.1 First-pass yield, eventual yield and rolled yield

A hypothetical route receives 1,000 units. Of these, 920 pass without rework, 50 initially fail but pass after authorized rework, and 30 are scrapped. The categories are mutually exclusive and reconcile to the input quantity.

$$\text{First-pass yield} = 920 / 1000 = 92.0\%$$

$$\text{Eventual yield} = (920 + 50) / 1000 = 97.0\%$$

The 97.0% value does not describe first-pass performance. Reporting only eventual yield would hide the 5.0% rework burden and the original failure history. Neither percentage establishes acceptance of an individual unit with an unresolved mandatory condition. For a second illustrative route, stage first-pass yields are 0.98, 0.97 and 0.99, each defined conditionally on units reaching that stage through the intended route.

$$\text{Rolled first-pass yield} = 0.98 * 0.97 * 0.99 = 0.941094$$

The resulting 94.1094% describes the model's through-route first-pass fraction. Use compatible populations and definitions; do not multiply unrelated dashboard percentages or combine reworked and unworked streams without explaining the accounting.

0.2 Capability and overall performance

Assume a synthetic dimensional specification from 9.90 to 10.10 mm, mean 10.04 mm, a justified within-subgroup standard-deviation estimate of 0.020 mm and an overall sample standard deviation of 0.030 mm.

$$C_p = (10.10 - 9.90) / (6 * 0.020) = 1.66667$$

$$C_{pk} = \min(0.06, 0.14) / (3 * 0.020) = 1.00000$$

$$P_p = 0.20 / (6 * 0.030) = 1.11111$$

$$P_{pk} = \min(0.06, 0.14) / (3 * 0.030) = 0.66667$$

These point estimates depend on the stated estimators and population. They do not supply an AF acceptance threshold. Investigate stability, distribution, dependence, sample adequacy and measurement effects before predictive interpretation. The larger overall variation may reflect between-group variation, drift or other conditions requiring explanation. A mean outside a specification can produce a negative centered index; it shall not be silently truncated to zero. A one-sided specification requires the appropriate one-sided expression. A high index does not release a known nonconforming item.

0.3 Control limits are not specification limits

For illustration, suppose a stable process model has known mean 10.000 mm and standard deviation 0.020 mm. Independent subgroups contain five items. A chosen three-standard-error chart gives:

Standard error = $0.020 / \sqrt{5} = 0.00894427 \text{ mm}$

Lower control limit = $10 - 3 * 0.00894427 = 9.97317 \text{ mm}$

Upper control limit = $10 + 3 * 0.00894427 = 10.02683 \text{ mm}$

A subgroup mean of 10.040 mm exceeds the upper control limit even though it lies inside the illustrative product specification of 9.90 to 10.10 mm. It signals a process-model concern; it does not by itself identify which individual products fail the specification. The example assumes known parameters and independent, suitably distributed observations. Estimated limits, variable subgroup size, autocorrelation or low-volume mixed production require an appropriate model. Three-sigma limits are an illustrative selection, not a universal AF requirement. Follow the approved reaction plan and assess the affected population.

O.4 Measurement uncertainty and a guarded decision

Three independent standard-uncertainty contributions are 0.002, 0.003 and 0.001 mm, already expressed at the output quantity. The illustrative budget has no covariance terms.

$uc = \sqrt{0.002^2 + 0.003^2 + 0.001^2} = 0.00374166 \text{ mm}$

$U = 2 * uc = 0.00748331 \text{ mm}$

The factor 2 is illustrative; its coverage interpretation requires justification. Suppose the approved rule accepts only when the complete interval from $y-U$ to $y+U$ lies within the inclusive 9.90 to 10.10 mm specification. It rejects when the complete interval is outside; other cases remain HOLD.

Table O-1. Illustrative guarded decisions

Result y	Upper interval endpoint	Outcome under the stated rule
10.089 mm	10.09648331 mm	PASS; the lower endpoint is also inside.
10.094 mm	10.10148331 mm	HOLD; the interval crosses the upper boundary.
10.109 mm	10.11648331 mm	FAIL; the lower endpoint is 10.10151669 mm, above the limit.

The second result is nominally inside the specification but not accepted by this rule. Use unrounded calculation values. The example does not prescribe one decision rule for every application.

O.5 A finite-lot zero-acceptance sampling plan

Consider a hypothetical lot of 1,000 items containing exactly 10 defective items. Select 100 items uniformly without replacement and accept only if no defect is observed, assuming perfect defect detection. The probability of accepting this particular lot is:

$P(\text{accept}) = C(990, 100) / C(1000, 100) = 0.34692771$

An independent Bernoulli approximation would give:

$P(\text{accept, approximation}) = 0.99^{100} = 0.36603234$

The difference arises because the actual sampling is without replacement from a finite population. The plan still accepts this 1%-defective lot about 34.69% of the time under the stated assumptions. Zero defects in the sample does not mean zero defects in the remaining lot. This is a model illustration, not a recommended plan. Real selection, lot homogeneity, inspection sensitivity and defect consequence require justification. The plan's risks shall be approved before sampling; failed samples shall not be replaced merely to obtain acceptance.

0.6 Zero observed defects and an upper bound

For 125 independent representative Bernoulli observations with zero observed defects, an exact one-sided 95% upper confidence bound on the defect probability under that model is:

$$p_{\text{upper}} = 1 - 0.05^{(1/125)} = 0.02368096$$

The bound is approximately 2.3681%, not zero. It describes the confidence procedure under the assumptions; it is not a 95% posterior probability statement about a fixed unknown value without an additional inferential model. Repeated tests of one item, common-cause dependence, selective exclusion and imperfect detection can invalidate the simple interpretation. A finite fixed lot sampled without replacement requires its own appropriate model. The calculation does not establish that a particular uninspected item is conforming.

0.7 Material reconciliation and genealogy

A synthetic receipt contains 1,000 kg. The store issues 640 kg and retains 360 kg. The issued quantity becomes 420 kg in product, 190 kg in chips, 20 kg in test specimens and 10 kg in documented process loss.

$$\text{Stock balance} = 640 + 360 = 1000 \text{ kg}$$

$$\text{Issued-material balance} = 420 + 190 + 20 + 10 = 640 \text{ kg}$$

This closes the declared mass account but does not by itself establish correct alloy identity or process conformity. Chips remain recovered material with their own identity and disposition; they do not automatically return to qualified input stock. If the recorded components sum to 635 kg, the unexplained difference is 5 kg. Investigate it against the approved measurement and reconciliation basis. Do not create a fictitious scrap transaction to force closure. Split and merge events must preserve every relevant parent even when the mass total balances.

0.8 Accumulated out-time and issue eligibility

A hypothetical material has an approved cumulative out-time allowance of 8 hours under defined conditions. Recorded exposures are 85, 110 and 95 minutes, all subject to the same accumulation rule.

$$\text{Consumed out-time} = 85 + 110 + 95 = 290 \text{ minutes}$$

$$\text{Remaining out-time} = 480 - 290 = 190 \text{ minutes}$$

A planned operation needs 150 minutes plus an approved 45-minute handling and contingency allowance, totaling 195 minutes. It exceeds the remaining allowance by 5 minutes and is not eligible under this basis. Returning the material to a freezer does not reset the 290 minutes unless a separate authorized technical basis explicitly permits it. Unknown exposure, changed temperature or a different life-clock definition requires assessment rather than automatic subtraction. These values are illustrative and shall not be used as material-specific storage instructions.

0.9 Effective equipment output and OEE

Assume a declared planned production period of 480 minutes, 60 minutes of downtime, an ideal cycle of 1 minute per unit, 360 units produced and 342 conforming units. All quantities refer to the same period and definition.

$$\text{Availability} = (480 - 60) / 480 = 0.875$$

$$\text{Performance} = (360 * 1) / 420 = 0.85714286$$

$$\text{Quality fraction} = 342 / 360 = 0.95$$

$$\text{OEE} = 0.875 * 0.85714286 * 0.95 = 0.7125$$

The equivalent calculation is 342 ideal-cycle minutes divided by 480 planned minutes, or 71.25%. This diagnostic does not establish that all 342 units are released; their individual mandatory evidence remains necessary. Declare planned-time exclusions, rework treatment and product mix. Do not improve the metric by excluding unplanned losses from the denominator or selecting an unrealistic

ideal cycle. Capacity planning shall also account for downstream inspection and release constraints.

O.10 A fixed-lead-time inventory illustration

Suppose independent daily demand has mean 40 eligible units and standard deviation 8 units. Lead time is fixed at 12 days. Under a justified normal approximation and a selected factor of 1.645:

Mean lead-time demand = $40 * 12 = 480$ units

Demand standard deviation = $8 * \text{sqrt}(12) = 27.71281$

Illustrative safety stock = $1.645 * 27.71281 = 45.58758$

Reorder point = $\text{ceil}(480 + 45.58758) = 526$ units

The factor, service interpretation and distribution assumptions require local approval. Variable lead time, correlated demand, minimum order quantities and shelf life change the model. Inventory position shall distinguish eligible stock, firm replenishment and commitments. Material on order is not physically available and remains subject to incoming acceptance. Quarantined or expired stock shall not be counted as usable buffer. The calculation is a planning illustration, not a supply-continuity guarantee.

O.11 Recovery gap and qualified alternatives

A hypothetical critical source has 240 usable units available and demand of 40 units per day. Its qualified stock coverage is 6 days. A demonstrated restoration route takes 9 days.

Coverage = $240 / 40 = 6$ days

Uncovered interval = $\max(0, 9 - 6) = 3$ days

Nominal gap = $3 * 40 = 120$ units

An alternate supplier promising delivery in 5 days is not a solution if an additional 6 days of necessary qualification must occur first; its earliest eligible availability is 11 days in this simple sequential case. Activities may overlap only where a technically justified plan supports the overlap. The example assumes constant demand and fully usable stock. Actual response shall include life, transport, inspection, approval and uncertainty. A continuity dashboard shall display the technical eligibility date rather than only the commercial delivery date.

O.12 Recycled-content allocation through yield loss

A synthetic batch uses 1,000 kg of input: 400 kg eligible recycled material and 600 kg other material. Output is 900 kg and documented loss is 100 kg. Under an expressly approved proportional-loss allocation:

Recycled input fraction = $400 / 1000 = 40.0\%$

Allocated recycled output = $900 * 0.40 = 360$ kg

Output fraction = $360 / 900 = 40.0\%$

Claiming $400/900 = 44.44\%$ would allocate all recycled input to output while assigning every loss to the other input, which is not the stated proportional model. A different allocation requires its own authorized evidence and transparent claim wording. Physical segregation, measured composition and accounting allocation are distinct. No quantity claim establishes aerospace material qualification, and the same environmental attribute shall not be counted again for another output without a valid accounting basis.