



AF-TR-9300-001

Quantitative Methods and Worked Calculations

Space / Aviation / Defense

FIRST EDITION

AF 9300 SUPPORTING DOCUMENT



Document identity and use

FIRST EDITION MANUSCRIPT - Prepared for Aerospace Federation review

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Authority	Publisher approval and effective date are not assigned. Local use requires authorized adoption and completed parameters.

Use with the main standard

This document implements the identified AF 9300 clauses and annexes. Read the complete applicable requirements in the main standard. The source anchors are navigation references; a template does not replace technical judgement, evidence, permission or eligible approval.

AF-ML-9300-001 lists the complete supporting set. AF-CM-9300-001 maps the main clauses and annexes to working documents. AF-TP-9300-001 establishes the common local parameter baseline; add the relevant discipline profiles.

Record control

Identify the object, configuration, record owner and dates. Reference attachments by stable identifier and version. Keep superseded evidence and decisions traceable. Use the approved local retention, access and authentication arrangements. No test level, numerical limit, interval or acceptance value is assigned by a blank template.

Implementation model: Translate the stated responsibilities and controls into the actual organization. Informative examples remain explanatory; use approved programme-specific values for operational decisions.

1 Use of quantitative examples

All values and cases in this companion are synthetic. They explain assumptions, calculations and limitations; they are not test levels, product evidence or mandatory default parameters. Select and approve the actual model and values before relying on a calculation.

2 Worked calculations

Status: Informative. All numbers, objects and criteria in this annex are synthetic. They are instructional examples, not default test levels, qualification margins, acceptance limits or evidence from an actual product.

Q.1 Combining an uncertainty budget

An illustrative measurement model adds four independent, zero-mean uncertainty contributions after applying known corrections. Repeated observations have standard deviation 0.60 units and nine independent observations are averaged. Calibration gives expanded uncertainty 0.80 units with factor 2. Resolution is 0.10 units. Residual drift is bounded by plus or minus 0.30 units with a rectangular distribution.

Table Q-1. Example standard uncertainty components

Contribution	Conversion	Standard uncertainty
Repeatability of mean	0.60 / sqrt(9)	0.20000 units
Calibration	0.80 / 2	0.40000 units
Resolution rounding	0.10 / sqrt(12)	0.02887 units
Residual drift	0.30 / sqrt(3)	0.17321 units

$$u_c = \sqrt{0.20^2 + 0.40^2 + 0.02887^2 + 0.17321^2}$$

$$u_c = 0.48045 \text{ units}; U = 2 * u_c = 0.96090 \text{ units}$$

The factor 2 is an assumed reporting choice for this example, not an automatically exact coverage probability. The calculation assumes independence and an adequate additive model. Shared calibration effects, influential nonlinearity or a different distribution would require a revised calculation. Averaging reduces the independent repeatability contribution; it does not divide the shared calibration contribution by sqrt(9).

Q.2 Correlation changes the result

Two input contributions to the same output have standard uncertainties 0.40 and 0.30 units, with unit sensitivities. If independent, the combined value is 0.50 units. With correlation coefficient 0.60, the covariance term is material.

$$u_c^2 = 0.40^2 + 0.30^2 + 2 * (0.60) * (0.40) * (0.30)$$

$$u_c = \sqrt{0.394} = 0.62769 \text{ units}$$

The correlated result is about 25.5% larger than the independent result. The correlation must arise from a justified common influence or data model; it must not be selected to force a desired outcome. For several inputs, the covariance model must be mathematically consistent and physically defensible. Listing instruments separately does not establish independence.

Q.3 A three-state upper-limit decision

The synthetic specification is y no greater than 100.00 units, with equality permitted. The agreed illustrative rule accepts when the complete interval is at or below the limit, rejects when the complete interval is above it, and otherwise reports indeterminate. Expanded uncertainty is 1.00 unit for these examples.

Table Q-2. Uncertainty-aware outcomes

Measured y	Interval $y \pm U$	Outcome
97.90	96.90 to 98.90	Conforming / PASS, subject to valid evidence.
99.60	98.60 to 100.60	Indeterminate / HOLD.
101.20	100.20 to 102.20	Nonconforming / FAIL.
101.00	100.00 to 102.00	Indeterminate: the interval touches the permitted boundary.

Accept if $y + U \leq 100.00$

Reject if $y - U > 100.00$; otherwise HOLD

A different agreed rule can produce a different decision and must be identified. The example does not claim a particular consumer-risk probability without an appropriate statistical model. Decisions use unrounded values; reported rounding must not change the outcome.

Q.4 Zero-failure independent-demand demonstration

Suppose independent identically distributed Bernoulli demands all succeed, the success probability is constant, the number n is fixed in advance and the claim concerns that defined demand population. With zero failures, a one-sided exact lower confidence bound on success probability is obtained from $R_L^n = \alpha$.

$$R_L = \alpha^{(1/n)}$$

$$n_{\text{required}} = \text{ceil}(\ln(\alpha) / \ln(R_{\text{target}}))$$

Table Q-3. Zero-failure sample sizes at 95% one-sided confidence

Target success probability	α	Minimum independent successes
0.90	0.05	29
0.95	0.05	59
0.99	0.05	299
0.999	0.05	2995

For 300 independent successes, R_L is approximately 0.990064. This is not proof of perfect reliability. Repeated demands on a changing or dependent system do not automatically satisfy the assumptions. A production-population claim also needs a sampling basis appropriate to that population. Failures, optional stopping or changed conditions require the corresponding analysis rather than this zero-failure formula.

Q.5 Zero-failure exposure under a constant-rate model

For a Poisson failure process with constant rate λ and total valid exposure T , the probability of zero failures is $\exp(-\lambda T)$. A one-sided upper rate bound after zero failures is:

$$\lambda_U = -\ln(\alpha) / T$$

$$T = 10000 \text{ h}, \alpha = 0.05: \lambda_U = 0.000299573 \text{ per h}$$

To demonstrate an upper rate no greater than 0.0001 per hour at the same confidence with zero failures, the model requires at least 29957.323 hours of valid exposure, rounded upward as appropriate to the exposure resolution. This is exposure accounting under a constant-rate assumption, not a universal test requirement.

Wear-out, changing stress, common causes, repair effects and incomplete detection can invalidate the model. The tested population and exposure conditions remain part of the claim. A failure-free chamber campaign does not establish the same rate in unrelated field conditions.

Q.6 PSD integration and RMS

For an illustrative one-sided acceleration PSD, assume a flat density of $0.020 \text{ g}^2/\text{Hz}$ from 20 to 100 Hz and $0.010 \text{ g}^2/\text{Hz}$ from 100 to 200 Hz, with zero outside the stated band. The mean-square acceleration is the area under the PSD.

$$a_{\text{rms}}^2 = 0.020 \cdot (100 - 20) + 0.010 \cdot (200 - 100) = 2.60 \text{ g}^2$$

$$a_{\text{rms}} = \sqrt{2.60} = 1.61245 \text{ g}$$

Table Q-4. Band contributions

Band	Density	Mean-square contribution
20-100 Hz	$0.020 \text{ g}^2/\text{Hz}$	1.60 g^2
100-200 Hz	$0.010 \text{ g}^2/\text{Hz}$	1.00 g^2
Total	Defined one-sided profile	2.60 g^2

This arithmetic does not establish an appropriate qualification environment. Different spectra can have the same RMS but excite different structural responses. Real spectra need appropriate integration of their actual shape, processing, units and frequency resolution.

Q.7 Decibels, amplitude and power

For compatible amplitude quantities with a common reference and the appropriate physical relationship, an amplitude ratio uses $20 \cdot \log_{10}(A_2/A_1)$. A power ratio, including a PSD ratio in compatible units, uses $10 \cdot \log_{10}(P_2/P_1)$.

$$\text{Amplitude ratio at } +6 \text{ dB} = 10^{(6/20)} = 1.99526$$

$$\text{Power or PSD ratio at } +6 \text{ dB} = 10^{(6/10)} = 3.98107$$

Multiplying an amplitude by 6 because the increase is 6 dB is incorrect. Doubling a PSD increases RMS by $\sqrt{2}$ over an unchanged band. The reference, impedance assumptions where relevant, and bandwidth must be stated. These relationships are mathematical conversions, not prescribed qualification margins.

Q.8 Exposure accounting after a control excursion

A synthetic specification requires 120 minutes of valid dwell after all defined article stability conditions are satisfied. A run log contains 18 minutes of approach, 52 minutes of valid dwell, 9 minutes outside tolerance and 68 further valid minutes after the required recovery check.

$$\text{Valid accumulated dwell} = 52 + 68 = 120 \text{ min}$$

Elapsed time in the listed sequence = 18 + 52 + 9 + 68 = 147 min

The 120-minute arithmetic closes the duration only if the approved specification permits accumulation across the interruption and the excursion does not invalidate the article or mechanism. If uninterrupted dwell was required, the two segments cannot simply be added. The 9-minute excursion requires an anomaly disposition even when sufficient valid dwell is later achieved.

Q.9 Guarded performance margin

A synthetic upper performance limit is 50.0 units. A valid result is 47.8 units with expanded uncertainty 1.2 units under the agreed interval rule. A conservative distance from the upper interval endpoint to the limit is:

$$M = \text{limit} - (y + U) = 50.0 - (47.8 + 1.2) = 1.0 \text{ unit}$$

The result has positive guarded distance for this criterion. It does not establish an input qualification margin, a reliability probability or a reserve against every failure mode. If U grows to 2.5 units after a justified uncertainty review, M becomes -0.3 units and the example rule no longer establishes PASS. Historical results near the boundary may therefore need reassessment after a metrology change.

Q.10 Illustrative cumulative-damage accounting

Assume, solely for arithmetic illustration, a linear damage model with two validated constant-amplitude reference lives: N1 = 20000 cycles and N2 = 10000 cycles. The article receives n1 = 4000 and n2 = 1000 cycles.

$$D = n1/N1 + n2/N2 = 4000/20000 + 1000/10000 = 0.30$$

The number 0.30 is the output of the assumed model. It is not proof that exactly 30% of real life has been consumed or that 70% remains. Sequence effects, scatter, mean load, environment and competing mechanisms can defeat this interpretation. A life claim requires a validated model, applicable data and the approved reliability/uncertainty treatment. The original exposures shall remain in the article history regardless of the model used.

Q.11 Model residuals and a local acceptance criterion

A synthetic validation dataset contains four observed outputs [100, 110, 90, 100] and predictions [101, 108, 93, 98]. Residuals, prediction minus observation, are [1, -2, 3, -2].

$$\text{Mean residual} = (1 - 2 + 3 - 2)/4 = 0$$

$$\text{RMSE} = \sqrt{(1^2 + (-2)^2 + 3^2 + (-2)^2)/4} = 2.12132$$

$$\text{Maximum absolute residual} = 3 \text{ units}$$

A zero mean residual does not imply accurate prediction at each condition. If the predeclared local criterion is an absolute residual no greater than 2 units, the third condition fails even though the mean bias is zero. Reference measurement uncertainty, domain coverage and data independence still require assessment. This example supports no claim beyond the synthetic dataset.

Q.12 Capability and coverage metrics without compensating scores

A programme has 80 applicable requirement conditions: 70 PASS, 3 FAIL and 7 HOLD, with 10 additional approved N/A conditions outside the applicable denominator. Its applicable PASS fraction is 70/80 = 87.5%. Counting N/A as PASS would misleadingly produce 80/90 = 88.9%.

The useful report gives all state counts, the denominator, critical conditions and their consequences. Neither percentage authorizes qualification while an essential condition is FAIL or HOLD. A coverage indicator is a management view of work, not a substitute for criterion-level technical decisions.